A JOINT MARKOV RANDOM FIELD/MARKED POINT PROCESS IMAGE MODEL UNDER THE BAYESIAN FRAMEWORK

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Presentation Outline

- Introduction: background and motivation
- A joint markov random field/marked point process model under the Bayesian framework
- Experimental results and comparison

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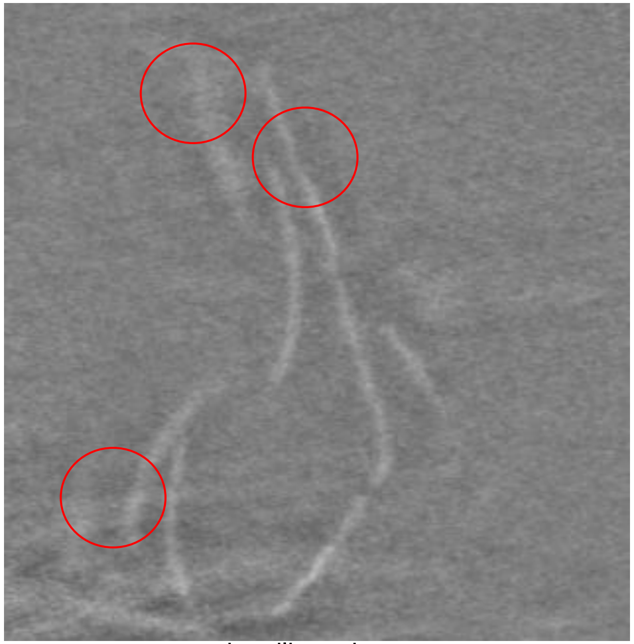
Background and motivation

$$V(w|y)=\sum_{w_i\in W}(V_o(y|w_i)+V_s(x_{w_i}^{MAP}))+\sum_{w_i\sim w_j}V_p(w_i,w_j)$$

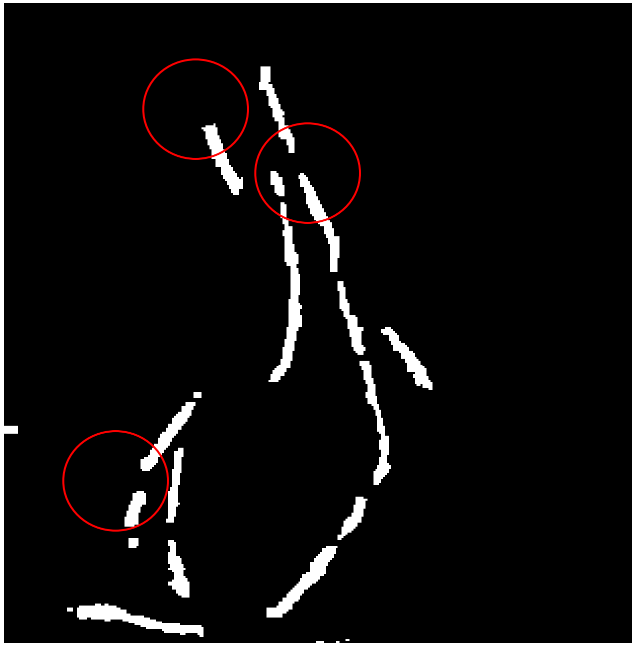
Object potential (MPP) — Segmentation potential (MRF) — Geometric prior (MPP)

The disadvantage of the old MPP/MRF model:

1. time consuming
2. artifacts along segmentation boundaries



the silicate image



the segmentation part of the old MPP/MRF model

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A joint markov random field/marked point process model under the Bayesian framework

denote w the object configuration, x the segmentation and y the observed image. The posterior probability of w, x given y is given by $p(w, x|y)$. According to the Bayes' rule, we have:

$$p(w, x|y) = \frac{f(y|w, x)p(w, x)}{f(y)}$$

$f(y|w, x)$: the conditional probability density function of y given w, x.

$p(w, x)$: the joint probability mass function (a joint prior).

$f(y)$: the probability density function of y.

Our target is to find the (w, x) that maximize the posterior probability $p(w, x|y)$. $f(y)$ does not depend on w or x.

A joint markov random field/marked point process model under the Bayesian framework

Since $f(y)$ does not depend on w or x, it is equivalent to obtain such a (w, x) that minimize an energy function.

$$V(w, x|y) = V_d(y|w, x) + V_p(w, x)$$

$$V_d(y|w, x) = -ln(f(y|w, x))$$

$$V_p(w, x) = -ln(p(w, x))$$

recall our old model:

$$V(w \mid y) = \sum_{w_i \in W} (V_o(y \mid w_i) + V_s(x_{w_i}^{MAP})) + \sum_{w_i \sim w_j} V_p(w_i, w_j)$$

Object potential (MPP)

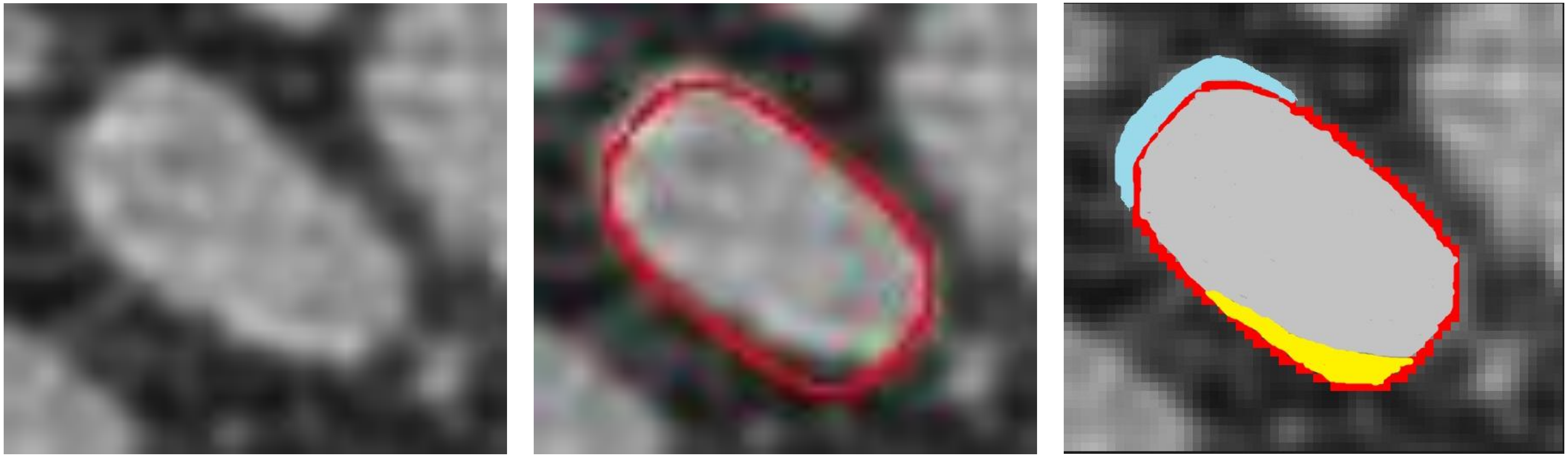
Segmentation potential (MRF)

Geometric prior (MPP)

A joint markov random field/marked point process model under the Bayesian framework

$f(y|w,x)$: the conditional probability density function of y given w, x.

in most cases, the MPP model cannot perfectly match particles' boundaries.



(a) a NiCrAl particle (b) a superellipse model (c) the mismatch between the MPP model and the boundary

A joint markov random field/marked point process model under the Bayesian framework

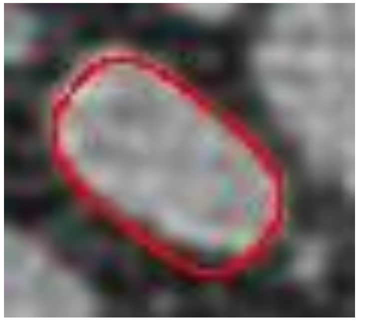
four kinds of pixels:

foreground pixels in object regions (grey)

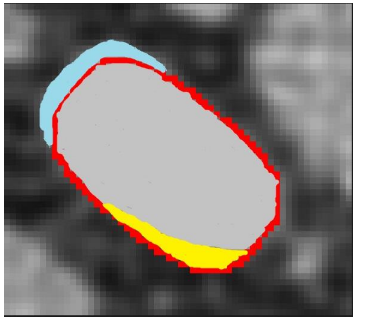
background pixels in object regions (yellow)

foreground pixels out of object regions (blue)

background pixels out of object regions



(b) a superellipse model



(c) the mismatch between the MPP model and the boundary

$$f(y|w,x) = \prod_{\substack{s \in \boldsymbol{S}_w \\ x_s = f}} \frac{\exp(-\frac{(y_s-\mu_f)^2}{2\sigma_f^2})}{\sqrt{2\pi\sigma_f^2}} \prod_{\substack{s \in \boldsymbol{S}_w \\ x_s = b}} \frac{\exp(-\frac{(y_s-\mu_b)^2}{2\sigma_b^2})}{\sqrt{2\pi\sigma_b^2}}$$

$$\prod_{\substack{s \in \boldsymbol{S}/\boldsymbol{S}_w \\ x_s = f}} \frac{\exp(-\frac{(y_s-\mu_f)^2}{2\sigma_f^2})}{\sqrt{2\pi\sigma_f^2}} \prod_{\substack{s \in \boldsymbol{S}/\boldsymbol{S}_w \\ x_s = b}} \frac{\exp(-\frac{(y_s-\mu_b)^2}{2\sigma_b^2})}{\sqrt{2\pi\sigma_b^2}}$$

Silhouette S_w: the projection of w onto the image region

A joint markov random field/marked point process model under the Bayesian framework

$p(w,x)$ is a joint probability mass function describing the interactions between neighboring pixel pairs (x_s, x_r) , object pairs (w_i, w_j) and the relation between the object field and the label field.

$$p(w,x) = \frac{1}{z}\exp(-\sum_{\{r,s\}\in\mathcal{C}} \beta_1 t_1(x_r, x_s) - \sum_{s\in S/S_w} \alpha_1 t_1(x_s, b) - \sum_{s\in S_w} \alpha_2 t_1(x_s, f) - \sum_{w_i\sim w_j} \beta_2 t_2(w_i, w_j))$$

$$t_1(a,b) = \begin{cases} 0 & \text{if } a = b \\ 1 & \text{if } a \neq b \end{cases}$$

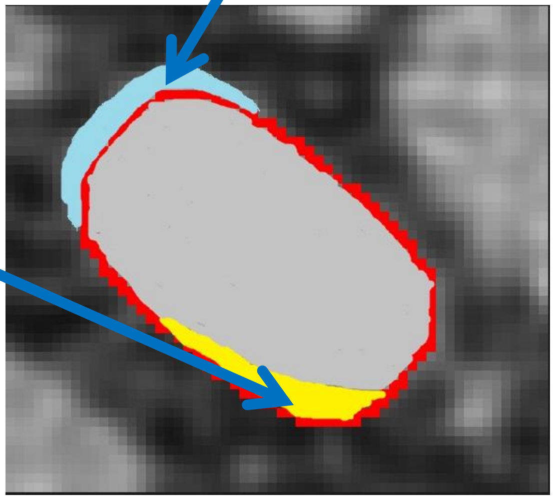
$t_2(w_i, w_j)$ is a function describing the interaction of object pairs.

A joint markov random field/marked point process model under the Bayesian framework

$$p(w,x) = \frac{1}{z}\exp(-\sum_{\{r,s\}\in C}\beta_1 t_1(x_r,x_s) - \sum_{s\in S/S_w}\alpha_1 t_1(x_s,b) - \sum_{s\in S_w}\alpha_2 t_1(x_s,f) - \sum_{w_i\sim w_j}\beta_2 t_2(w_i,w_j))$$

background pixels in object regions (yellow)

foreground pixels out of object regions (blue)



(c) the mismatch between the MPP model and the boundary

Compare two models

new model: $$V(w,x|y) = V_d(y|w,x) + V_p(w,x)$$

$$\begin{aligned}
V_d(y|w,x) &= -ln(f(y|w,x)) \\
&= \sum_{\substack{s\in S_w \\ x_s=f}} [\frac{(y_s-\mu_f)^2}{2\delta_f^2} + \sqrt{2\pi\delta_f^2}] \\
&+ \sum_{\substack{s\in S_w \\ x_s=b}} [\frac{(y_s-\mu_b)^2}{2\delta_b^2} + \sqrt{2\pi\delta_b^2}] \\
&+ \sum_{\substack{s\in S/S_w \\ x_s=f}} [\frac{(y_s-\mu_f)^2}{2\delta_f^2} + \sqrt{2\pi\delta_f^2}] \\
&+ \sum_{\substack{s\in S/S_w \\ x_s=b}} [\frac{(y_s-\mu_b)^2}{2\delta_b^2} + \sqrt{2\pi\delta_b^2}]
\end{aligned}$$

$$\begin{aligned}
V_p(w,x) &= -ln(p(y,x)) \\
&= \sum_{\{r,s\}\in\mathcal{C}} \beta_1 t_1(x_r,x_s) \\
&+ \sum_{s\in S/S_w} \alpha_1 t_1(x_s,b) \\
&+ \sum_{s\in S_w} \alpha_2 t_1(x_s,f) \\
&+ \sum_{w_i\sim w_j} \beta_2 t_2(w_i,w_j)
\end{aligned}$$

old model: $$V(w\,|\,y) = \sum_{w_i\in W} (V_o(y\,|\,w_i) + V_s(x_{w_i}^{MAP})) + \sum_{w_i\sim w_j} V_p(w_i,w_j)$$

The optimization method

$$V(w,x|y) = V_d(y|w,x) + V_p(w,x)$$

$$V_d(y|w,x) = -ln(f(y|w,x))$$

$$V_p(w,x) = -ln(p(w,x))$$

Alternating Minimization algorithm[1]:

Initialization: $w^0 = \emptyset,\ x^0 = \underset{x}{\operatorname{argmin}} V_d(w^0, x) + V_p(w^0, x);$

for $k = 1, \ldots, n$ **do**

$\quad w^{k+1} = \underset{w}{\operatorname{argmin}}(V_d(w, x^k) + V_p(w, x^k));$

$\quad x^{k+1} = \underset{x}{\operatorname{argmin}}(V_d(w^{k+1}, x) + V_p(w^{k+1}, x));$

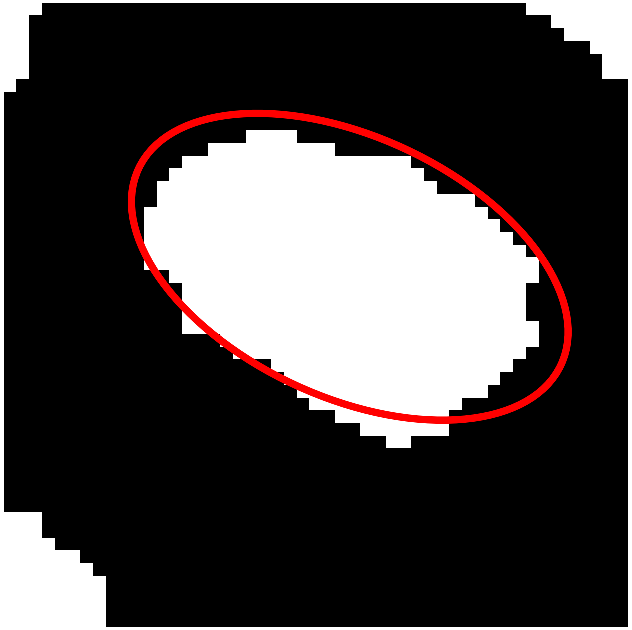
[1]I. Csisz`ar and G. Tusn`ady. “Information geometry and alternating minimization procedures,” *Statistics and decisions, Supplement Issue (1), pp.* 205-237, 1984.

$$w^{k+1} = \text{argmin}(V_d(w, x^k) + V_p(w, x^k))$$

This corresponds to a MPP model, we need to calculate the energy increase and decrease of adding and deleting an object .

Energy change for adding an object:

$$V_{w_i}^{add} = \sum_{s \in S_{w_i}} (V_{add}(s, x_s^{k-1} = b) + V_{add}(s, x_s^{k-1} = f)) + \sum_{\substack{w_j \in S_w \\ w_i \sim w_j}} \beta_2 t_2(w_i, w_j)$$



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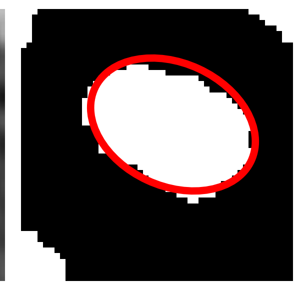
$$p(w, x) = \frac{1}{z} \exp(- \sum \beta_1 t_1(x_r, x_s) - \sum \alpha_1 t_1(x_s, f)$$

background pixels out of object regions => background pixels in object regions
or foreground pixels in object regions

$$V_{add}(s, x_s^{k-1} = f) = -\alpha_1$$

$$V_{add}(s, x_s^{k-1} = b) = min[E_{\text{b\_to\_f}}(s), \alpha_2]$$



foreground pixels out of object regions => foreground pixels in object regions

$$E_{\text{b\_to\_f}}(s) = \frac{\;}{2\sigma_f^2} + \ln\sqrt{2\pi\sigma_f^2} - \frac{\;}{2\sigma_b^2} - \ln\sqrt{2\pi\sigma_b^2}$$

$$+ \sum_{\{r,s\} \in C} \beta_1(t_1(x_r, f) - t_1(x_r, b))$$

$$w^{k+1} = \text{argmin}(V_d(w, x^k) + V_p(w, x^k))$$

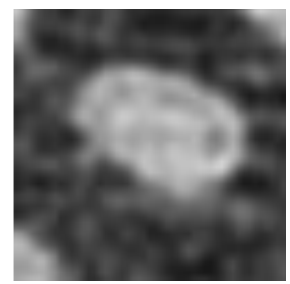
Energy change for deleting an object:

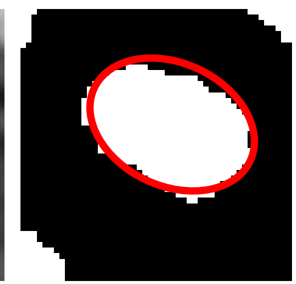
$$V_{w_i}^{del} = \sum_{s \in S_{w_i}} (V_{del}(s, x_s^{k-1} = b) + V_{del}(s, x_s^{k-1} = f)) - \sum_{\substack{w_j \in S_w \\ w_i \sim w_j}} \beta_2 t_2(w_i, w_j)$$

$$p(w,x) = \frac{1}{z}\exp(-\sum_{\{r,s\}\in C} \beta_1 t_1(x_r, x_s) - \sum_{s \in S/S_w} \alpha_1 t_1(x_s, f) - \sum_{s \in S_w} \alpha_2 t_1(x_s, b) - \sum_{w_i \sim w_j} \beta_2 t_2(w_i, w_j))$$

$$V_{del}(s, x_s^{k-1} = b) = -\alpha_2$$

$$V_{del}(s, x_s^{k-1} = f) = min[E_{\text{f\_to\_b}}(s), \alpha_1]$$





$$E_{\text{f\_to\_b}}(s) = \frac{(y_s - \mu_b)^2}{2\sigma_b^2} + ln\sqrt{2\pi\sigma_b^2} - \frac{(y_s - \mu_f)^2}{2\sigma_f^2} - ln\sqrt{2\pi\sigma_f^2} + \sum_{\{r,s\}\in C} \beta_1(t_1(x_r, b) - t_1(x_r, f))$$

The optimization method

$$V(w, x|y) = V_d(y|w, x) + V_p(w, x)$$

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$$V_p(w, x) = -ln(p(w, x))$$

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[1]I. Csisz`ar and G. Tusn`ady. "Information geometry and alternating minimization procedures," *Statistics and decisions, Supplement Issue (1), pp.* 205-237, 1984.

$$x^{k+1} = \underset{x}{\operatorname{argmin}}(V_d(w^{k+1}, x) + V_p(w^{k+1}, x))$$

This corresponds to a MRF model. We can easily get this via Graph Cuts.

edge	weight(cost)	for
$\{s, r\}$	β_1	$\{s, r\} \in \mathcal{C}$
$\{s, P\}$	$\frac{(y_s - \mu_b)^2}{2\sigma_b^2} + ln\sqrt{2\pi\sigma_b^2} + \alpha_2$	$s \in \boldsymbol{S}_{w^k}$
	$\frac{(y_s - \mu_b)^2}{2\sigma_b^2} + ln\sqrt{2\pi\sigma_b^2}$	$s \in \boldsymbol{S}/\boldsymbol{S}_{w^k}$
$\{s, Q\}$	$\frac{(y_s - \mu_f)^2}{2\sigma_f^2} + ln\sqrt{2\pi\sigma_f^2}$	$s \in \boldsymbol{S}_{w^k}$
	$\frac{(y_s - \mu_f)^2}{2\sigma_f^2} + ln\sqrt{2\pi\sigma_f^2} + \alpha_1$	$s \in \boldsymbol{S}/\boldsymbol{S}_{w^k}$

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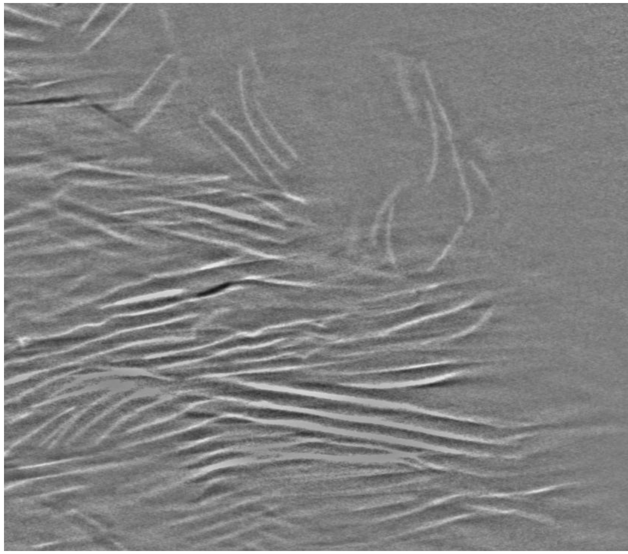
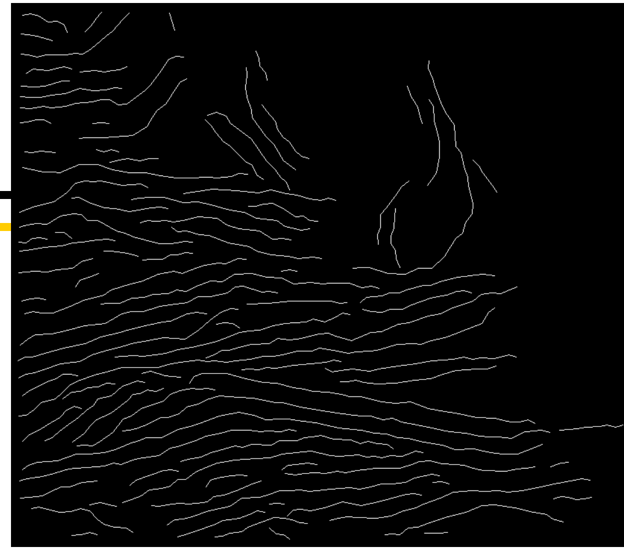
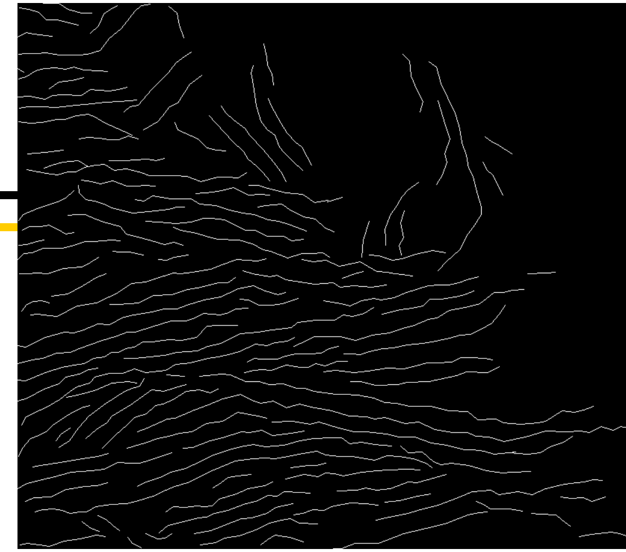


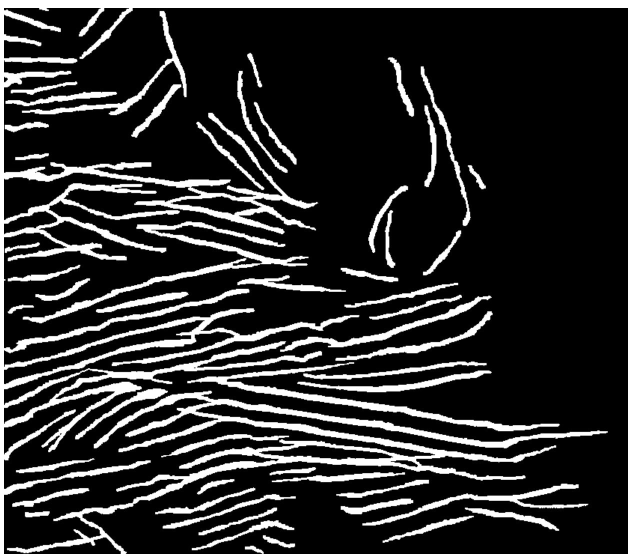
silicate image



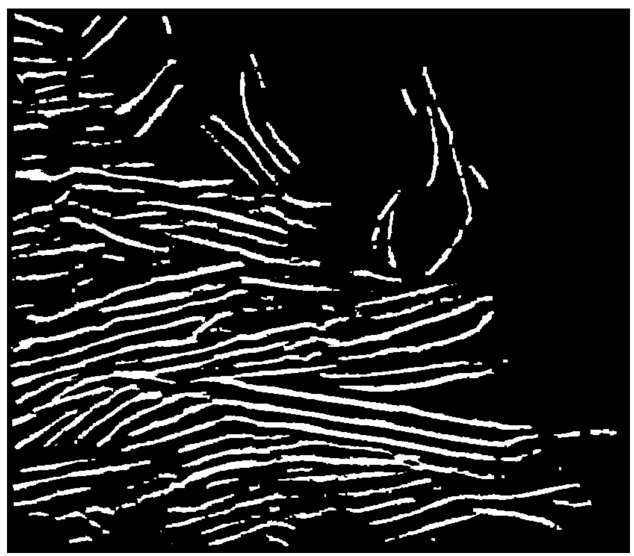
previous model
(object detection part)



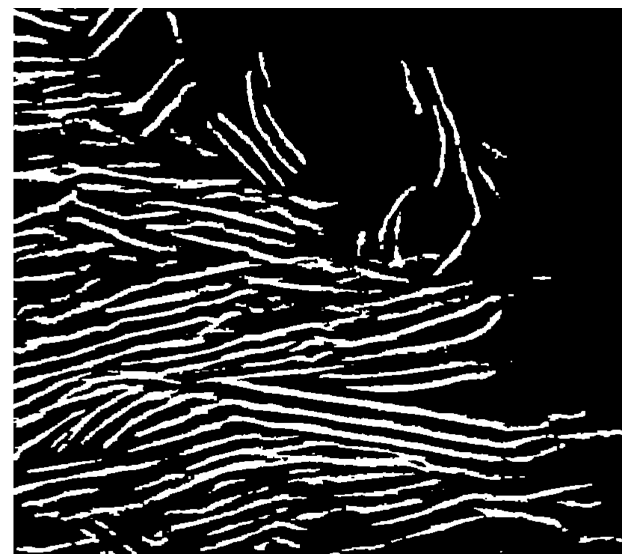
new model
(object detection part)



hand made segmentation

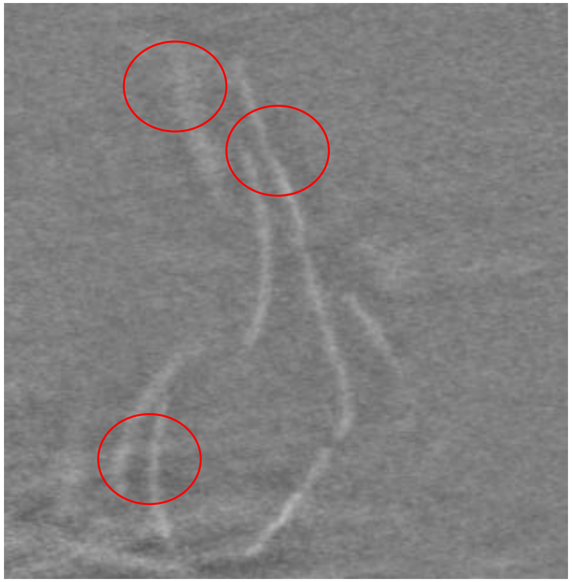


previous model
(segmentation part)



new model
(segmentation part)

Result comparison between old model and new model



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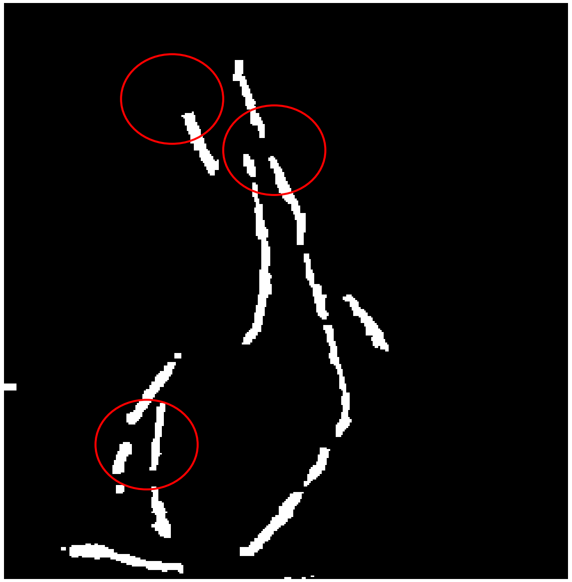
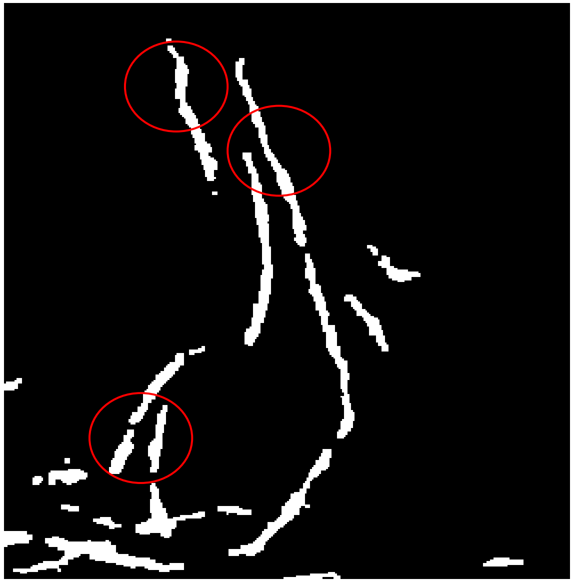


the segmentation part of the old MPP/MRF model



the segmentation part of the new MPP/MRF model

Result comparison between old model and new model

Type I error: the ratio of misclassified foreground pixels

Type II error: the ratio of misclassified background pixels

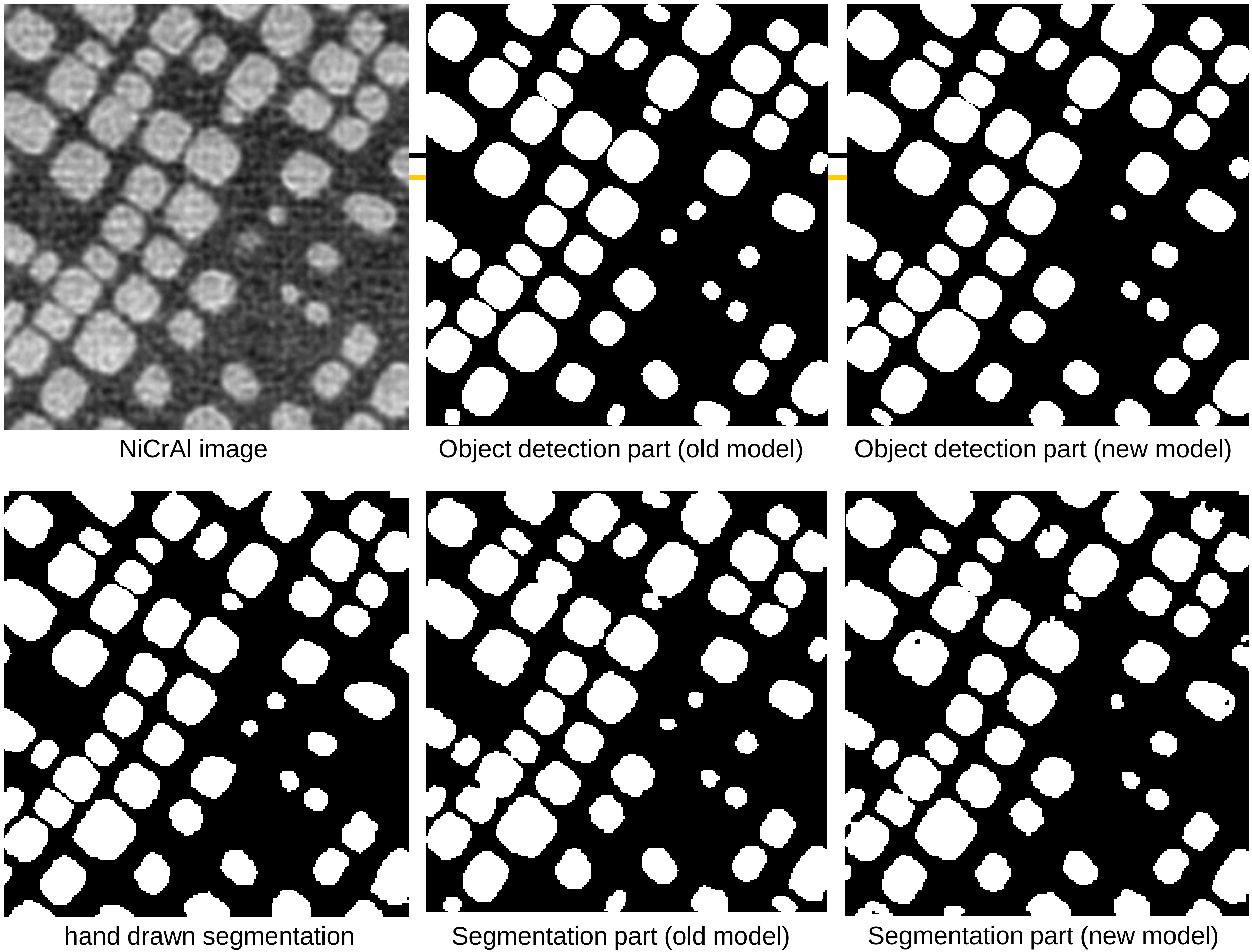
silicate image	Type I	Type II
old model	24.60%	1.30%
new model	15.93%	3.34%

Missed detection rate: the ratio of missed objects

False detection rate: the ratio of falsely detected objects

silicate image	M.R.	F.R.
old model	10.36%	3.58%
new model	5.18%	4.87%

Image size 834x729	old model	new model
computational time	6333.973s (105min)	2001.100s (33 min)


NiCrAl image
Object detection part (old model)
Object detection part (new model)
hand drawn segmentation
Segmentation part (old model)
Segmentation part (new model)

Result comparison between old model and new model

Type I error: the ratio of misclassified foreground pixels

Type II error: the ratio of misclassified background pixels

NiCrAl particle	Type I	Type II
old model	3.65%	3.53%
new model	6.10%	1.08%

Missed detection rate: the ratio of missed objects

False detection rate: the ratio of falsely detected objects

NiCrAl particle	M.R.	F.R.
old model	0.00%	1.96%
new model	1.96%	0.00%

Image size 236x248	old model	new model
computational time	1078.250s (17.9 min)	289.037s (4.8 min)

Thank you!